ANISOTROPIC THERMODYNAMICS AND $T/\sqrt{H}$ SCALING OF d-WAVE SUPERCONDUCTORS IN THE VORTEX STATE.

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The density of states of a 2D d-wave superconductor in the vortex state with applied magnetic field $\mathbf{H}$ in the plane is shown to exhibit fourfold oscillations as a function of the angle of the field with respect to the crystal axes. We find further that the frequency dependence of the density of states and the temperature dependence of transport coefficients obey different power laws, thus leading to different magnetic scaling functions, for field in the nodal and anti-nodal direction. We discuss the consequences of this anisotropy for measurements of the specific heat.

Some of the first results supporting the d-wave symmetry of the energy gap in the hole-doped high-T_c cuprates came from measurements of the specific heat in the vortex state [1]. This has triggered a renewed interest in the properties of unconventional superconductors in the mixed state [2,3,4].

Volovik[5] showed that in a magnetic field $H \ll H_{c2}$ the density of states (DOS) of a superconductor with lines of nodes is dominated by the extended quasiparticle states, rather than by the bound states in the vortex cores as for an s-wave gap. In a semiclassical treatment, the energy of a quasiparticle with momentum $\mathbf{k}$ at position $\mathbf{r}$ is shifted by $\delta\omega_{\mathbf{k}}(\mathbf{r}) = \mathbf{v}_s \cdot \mathbf{k}$, where $\mathbf{v}_s(\mathbf{r})$ can be approximated by the velocity field around a single vortex. Physical quantities then depend on $\mathbf{r}$ and have to be averaged over a unit cell of the vortex lattice. This method has been successful in describing the thermal and transport properties of the vortex state with the field $\mathbf{H}$ perpendicular to the CuO_2 layers [5,6,7,8,9].

Recently we have argued that the same approach remains qualitatively valid in $YBa_2Cu_3O_{7-\delta}$ and other not too anisotropic materials for $\mathbf{H}$ parallel to the layers and have computed the DOS[10]. We assume an order parameter $\Delta_{\mathbf{k}} = \Delta_0 \cos 2\phi$ over a cylindrical Fermi surface parameterized by the angle ϕ. The field $\mathbf{H}$ is applied in the a-b plane, at an angle α to the x-axis. The flow field of a vortex, $\mathbf{v}_s$, is elliptical due to the anisotropy of the penetration depth, $\varepsilon = \lambda_c/\lambda_{ab}$, but after the rescaling of the c-axis $\mathbf{v}_s = \hbar\hat{\beta}/2mr$ is isotropic.[10] Here r is the distance from the center of the vortex, and $\hat{\beta}$ is a unit vector along the current. Approximating the unit cell of the vortex lattice by a circle of radius $R = a^{-1}\sqrt{\Phi_0/\pi H}$, where $a \sim 1$ is a geometric constant, we obtain $\delta\omega_{\mathbf{k}}(\mathbf{r}) = E_H \sin\beta \sin(\phi - \alpha)/\rho$, where $\vec{\rho} = \mathbf{r}/R$ and a typical magnetic energy

is defined as $E_H = av^* \sqrt{\pi H/\Phi_0}/2$. In London theory $v^* = v_f/\sqrt{\varepsilon}$, where v_f is the Fermi velocity in the plane.

For undoped $YBCO$, $\varepsilon \approx 5.3$[3], and the anisotropy increases with deoxygenation[11]. Taking $v_f = 1.2 \times 10^7$cm/s[4] we estimate $E_H \approx 7.9a\sqrt{H}$ KT$^{-1/2}$ for the undoped samples, while for $\mathbf{H}\|c$ $E_H^c \equiv (v_F/v^*)E_H \approx 18.2a\sqrt{H}$ KT$^{-1/2}$. For in-plane field $\delta\omega_{\mathbf{k}} \propto \sin(\phi-\alpha)$ and the DOS $N(\omega,\alpha)$ depends on the angle between the field and the direction of the gap nodes, giving[12,10] $N(0,\alpha)/N_0 = 2\sqrt{2}E_H \max(|\sin\alpha|, |\cos\alpha|)/(\pi\Delta_0)$, where N_0 is the normal state DOS. Note that $N(0,\alpha) \propto \sqrt{H}$. For $\omega, E_H \ll \Delta_0$ the density of states $N(\omega,\alpha) \simeq (N_1(\omega,\alpha) + N_2(\omega,\alpha))/2$, where

$$\frac{N_i(\omega,\alpha)}{N_0} = \begin{cases} \frac{\omega}{\Delta_0}\left(1 + \frac{1}{2x^2}\right), & \text{if } x = \omega/E_i \geq 1; \\ \frac{E_i}{\pi\Delta_0 x}\left[(1+2x^2)\arcsin x + 3x\sqrt{1-x^2}\right], & \text{if } x \leq 1, \end{cases} \tag{1}$$

for $i = 1, 2$, $E_1 = E_H|\sin(\pi/4-\alpha)|$ and $E_2 = E_H|\cos(\pi/4-\alpha)|$.[10] In particular

$$\frac{N(\omega,\alpha)}{N_0} \approx \begin{cases} \frac{2\sqrt{2}E_H}{\pi\Delta_0}\left(1 + \frac{1}{3}\frac{\omega^2}{E_H^2}\right) & \text{for } \alpha = 0; \\ \frac{2E_H}{\pi\Delta_0} + \frac{\omega}{2\Delta_0} & \text{for } \alpha = \pi/4 \end{cases}. \tag{2}$$

The frequency dependence of $N(\omega,\alpha)$ follows different power laws for the field along a node or an anti-node, and consequently the specific heat coefficient C/T, NMR relaxation time T_1T and other quantities exhibit fourfold oscillations and a T or T^2 behavior depending on the direction of the field.

We now compute the low temperature specific heat $C(T,H)$ as in Ref.[6], and show the result in Fig. 1a. Here $\gamma_n = \pi^2 N_0/3$, and we have used $\varepsilon = 7$ and $E_H = 0.1\Delta_0$. For $a = 1$ and $\Delta_0 = 200$K this corresponds to $H \simeq 6.5T$. Taking[1] $\gamma_n = 20$mJ/mol K^2, the amplitude of the oscillations in $C/\gamma_n T$ for $\mathbf{H}\|ab$ at $T = 0$ is 0.5 mJ/mol K^2, close to a previous estimate[12]. This amplitude is reduced as T increases: at $T = 0.01\Delta_0 \simeq 2$K, it is 40% of the $T = 0$ value. This can explain why the oscillations have not been found in the one measurement done for two orientations of the field[1]. In an orthorhombic system the induced s-wave component of the gap shifts the position of the DOS minimum away from the $\pi/4$ direction, and in a heavily twinned crystal, such as used in Ref.[1], this further suppresses the amplitude of the oscillations.

For $T \ll E_H$, C/T varies as T and T^2 for $\mathbf{H}\|node$ and $\mathbf{H}\|antinode$ respectively. There exists a regime $E_H \ll T \ll E_H^c$ where the anisotropy is washed out, $C(\mathbf{H}\|ab)/T \propto T$ but $C(\mathbf{H}\|c)/T \simeq const$. This observation can help resolve some of the disagreement between the specific heat data obtained in Refs.[1,2,3]. The results of measurements both on single crystals[1] with $\mathbf{H}\|c$, and on polycrystalline samples[2] are well described by $C/T \propto \sqrt{H}$. Note that due to

large anisotropy, the supercurrents are nearly in the a-b plane for almost all orientations of the grains with respect to $\mathbf{H}$[13], so that both experiments effectively measure $C(\mathbf{H}\|c)$. Since the measured specific heat is a sum of the DOS dependent and "background" contributions the analysis is rather involved. Instead, Revaz *et al.*[3] analyzed the anisotropy $\delta C = C(\mathbf{H}\|c) - C(\mathbf{H}\|antinode)$, interpreting it as a pure vortex quantity. They found $\delta C/T$ temperature dependent, which can be understood since it becomes $T-$dependent for $E_H^c \gg T \geq E_H$.

We now define $C/(TE_H) \equiv N_0 F_C(X)/\Delta_0$, where $X = T/E_H$ is the scaling variable[14]. In the limit $X \gg 1$ we have $F_C(X) = 9\zeta(3)X + \ln 2/2X$, similar to the result for $\mathbf{H}\|c$[15]. In the opposite limit

$$F_C(X) = \begin{cases} 2\pi/3 + 9\zeta(3)X/2, & X \ll 1,\ \mathbf{H}\|node, \\ 2\sqrt{2}\pi/3 + 14\sqrt{2}\pi^3 X^2/45, & X \ll 1,\ \mathbf{H}\|antinode. \end{cases} \tag{3}$$

$F_C(X)$ is shown in Fig.1b for $\varepsilon = 5.3$. For $\mathbf{H}\|c$ the crossover from $C/TE_H \sim$ *const* occurs at $X_c \sim 0.5$, which was estimated [2] to be at $T/\sqrt{H} = 6.5$ $\mathrm{KT}^{-1/2}$, yielding $E_H^c \approx 30\sqrt{H}\mathrm{KT}^{-1/2}$. The crossover from small to large X in $\delta C(T,H)$, occurs at $X_{ab} \sim 0.15$, but the predicted $X \ll 1$ behavior was not found above[3] $T/\sqrt{H} \approx 0.55\mathrm{KT}^{-1/2}$, which implies $E_H \leq 4\sqrt{H}$ $\mathrm{KT}^{-1/2}$. Note that the interpolation used in Ref.[3] gives a linear correction in X, rather than quadratic as in Eq.(3), to the behavior at $X \ll 1$, which has led to an underestimate of the crossover scale. Notice also that the crossover in δC extends over a decade in X.

The experimental values for E_H^c and E_H are within a factor of 2 of our estimates, and E_H^c/E_H is then about 3 times the predicted ratio of $\sqrt{\varepsilon}$. These are quantitative rather than qualitative differences, given the roughness of the estimates and the experimental uncertainties. Also the constant a in general is not the same for $\mathbf{H}\|ab$ and $\mathbf{H}\|c$. Note that for Josephson coupled planes, the anisotropy in the the vortex lattice constants is H-dependent and differs from ε[16]. Finally, since the elastic moduli of the vortex lattice differ for $\mathbf{H}\|ab$ and $\mathbf{H}\|c$, the lattice contributes to δC[17,18,19].

Qualitative agreement between our estimates and the experimental results suggests that the angular oscillations of the specific heat are accessible at $T \leq 2\mathrm{K}$ and $H \sim 10-20\mathrm{T}$. Such measurements would be a simple bulk probe of gap symmetry allowing one to map out the position of the gap nodes.

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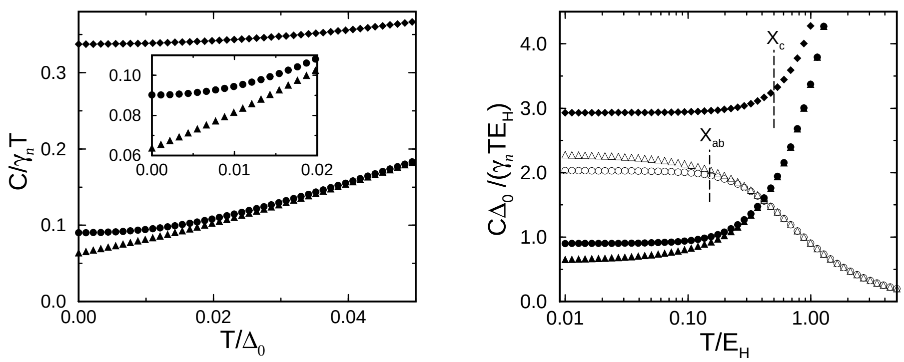

Figure 1: a)Normalized specific heat for $\mathbf{H}\|c$ (diamonds), $\mathbf{H}\|antinode$ (circles), and $\mathbf{H}\|node$ (triangles), for $\varepsilon = 7$ ($\delta \simeq 0.05$) and $E_H = 0.1\Delta_0$; Inset: low-T anisotropy; b) Scaling function for $C(\mathbf{H}\|c)$ (diamonds), $C(\mathbf{H}\|antinode)$ (full circles), $C(\mathbf{H}\|node)$ (full triangles), $\delta C(\mathbf{H}\|antinode)$ (open circles), and $\delta C(\mathbf{H}\|node)$ (open triangles).

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